Learning Control Theory for Dynamical Systems(Mathematical Theory of Control and Systems)

AUTHOR(S):

Arimoto, Suguru; Kawamura, S.; Miyazaki, F.

CITATION:

Arimoto, Suguru ...[et al]. Learning Control Theory for Dynamical Systems(Mathematical Theory of Control and Systems). 数理解析研究所講究録 1985, 562: 285-311

ISSUE DATE:

1985-05

URL:

http://hdl.handle.net/2433/99045

数理解析研究所講究録
562巻 1985年 285-311

Learning Control Theory for Dynamical Systems

Suguru Arimoto, S. Kawamura, and F. Miyazaki
(有本 卓) (川村 貞夫) (宮崎 文夫)
Faculty of Engineering Science, Osaka University

Abstract: Three types of learning control laws are proposed for mechanical or mechatronics systems with linear and non-linear dynamics, which may be operated repeatedly at low cost. Given a desired output y_d over a finite time duration [0,T] and an appropriate input u_o, these laws are formed by the following simple iterative processes: 1) $u_{k+1}= u_k+ \Phi(y_d-y_k)$, 2) $u_{k+1}= u_k + \Gamma\frac{d}{dt}(y_d-y_k)$, and 3) $u_{k+1}= u_k+ (\Phi+ \Gamma\frac{d}{dt})(y_d-y_k)$, where $u_k(u_{k+1})$ denotes the kth(k+1th) input, y_k the measured output at the kth operation corresponding to u_k, and Φ and Γ positive definite constant gain matrices. It is shown that the first law 1) with an appropriate gain matrix Φ is convergent in the sense that $y_k(t)$ approaches $y_d(t)$ as $k\to\infty$ uniformly in $t\in[0,T]$ if the objective system is linear and strictly positive. The same conclusion is also proved when the system is subject to a linear mechanical system. In addition, a rough sketch of the convergency proof of the second and third learning control laws is presented for a class of linear and nonlinear dynamical systems. Finally some discussions on potential applicabilities of these learning methods for robot controls are given.

:. INTRODUCTION

In the control engineering world little attention has been ›aid to "learning" since more than twenty years ago "perceptron" 'as introduced by Rosenblatt [1] in the field of pattern recogni- .ion and subsequently "learning systems" was discussed by Tsypkin 2]. This might be due to lack of dynamic structures in their .athematical models, since a finite number of only coefficient pa- 'ameters are modified through a learning process. However, is the oncept of "learning" still useless and irrelevant to control engi- .eering in the present robotic age or the forthcoming VLSI era? s a robot or any automatic machine unable to learn, without help of uman operators, anything from previous operation data and improve he performance of next operation, no matter how fortunate it is that variety of mechanical systems like robots can be operated repeatedly t low cost?

Taking these points into consideration, the authors [1]-[4] ecently proposed a learning control method based on a simple itera- ive algorithm of learning process for motion control of robots. his algorithm was called the "betterment process", which updates he control input based on the previous operation data and betters he performance of next operation in a certain sense, provided a esired motion trajectory is given. In the present paper we propose hree types of learning control scheme including the previous one or linear and nonlinear dynamical systems and prove their con- ergency under certain assumptions of the objective system dynamics. he potentialities of these proposed learning control methods in pplications of robot controls are also discussed.

The argument of this paper is developed in agreement with the principle that mathematical sophistication for the proof of effectiveness of the learning control law may be permissive but sophistication of control algorithms must not be permitted. The propsed methods together with the proof of their effectiveness were found in line with the engineering rule of thumb that the simpler the control algorithm, the more robust it is.

II. PROPOSAL OF THREE TYPES OF LEARNING CONTROL SCHEME

Suppose that a finite-dimensional dynamical system is given, which has control input vector u with dimension m and measurement output vector y with the same dimension, m. Then, consider a situation that given a desired output $y_d(t)$ over a finite-time interval [0, T], we are requested to find a control input $u^*(t)$, which excites the system and eventually produces an output $y^*(t)$ so that $y^*(t)$ must be coincident with $y_d(t)$ over $t \in [0, T]$. If a full description of the system is available, it may be possible to construct such a control input on the basis of system inversion techniques or so. However, in practical situations it is usual that only a rough sketch of the system is available or system parameters have inaccuracies to some extent even if a description of the system dynamics is known. In such a situation, it is already shown by the authors [1], [2], [3] that a learning control scheme called "betterment process" becomes quite effective. This learning control method can be described by a simple iterative rule of input modification defined as

$$u_{k+1}(t) = u_k(t) + \Gamma\frac{d}{dt}\{y_d(t) - y_k(t)\} \qquad (1)$$

which is schematically shown in Fig.1. Here, Γ is an $m \times m$ constant matrix called "gain matrix". In other words, at the k+1th run, the control input is updated in such a way that the previous input at the kth run is modified by being added a derivative of the error between the previous output $y_k(t)$ and the given desired output $y_d(t)$. The modification rule may be technically realized by a digital computation method illustrated in Fig.2.

By use of present VLSI technology, it is possible to assume that the continuous-time functions $y_d(t)$, $y_k(t)$, and $u_k(t)$ are sampled sufficiently densely, converted from analogue to digital, and stored in RAMs. Namely, it is assumed that those original time-continuous signals can be promptly reproduced from the memory with sufficiently high accuracy.

In addition to the iterative rule defined by Eq.(1), we introduce another simple iterative rule called and defined by

1) C^o-type Betterment Process (see Fig.3):

$$u_{k+1}(t) = u_k(t) + \Phi e_k(t), \tag{2}$$

where

$$e_k(t) = y_d(t) - y_k(t) \tag{3}$$

and Φ is an $m \times m$ constant gain matrix. In relation to this, we call the process of Eq.(1)

2) C^1-type Betterment Process:

$$u_{k+1}(t) = u_k(t) + \Gamma \frac{d}{dt} e_k(t). \tag{4}$$

In comparison with these we sometime discuss

3) Mixed-type Betterment Process (see Fig.4):

$$u_{k+1}(t) = u_k(t) + (\Phi + \Gamma \frac{d}{dt}) e_k(t). \tag{5}$$

III. CONVERGENCE OF C^o-TYPE BETTERMENT PROCESS FOR LINEAR TIME-INVARIANT SYSTEMS

In this section we will present a proof of convergence of a C^o-type betterment process provided the objective system is linear, time-invariant, and strictly positive.

Suppose that the system is subject to

$$y(t) = g(t) + \int_0^t H(t-\tau)u(t)d\tau, \quad t\varepsilon[0, T]. \tag{6}$$

We assume that at every operation trial, g(t) and H(t) are the same, namely,

$$y_k(t) = g(t) + \int_0^t H(t-\tau)u_k(\tau)d\tau, \quad t\varepsilon[0, T] \tag{7}$$

If the system has a state-space representation

$$\begin{cases} \dot{x} = Ax + Bu, \\ y = Cx, \end{cases} \tag{8}$$

the condition of Eq.(7) is fulfilled, provided every initial state $x_k(0)$ is set at the same fixed state, x^o, in every run. In fact, it is evident that

$$\begin{aligned} y_k(t) &= Ce^{At}x_k(0) + \int_0^t Ce^{A(t-\tau)}Bu_k(\tau)d\tau \\ &= Ce^{At}x^o + \int_0^t Ce^{A(t-\tau)}Bu_k(\tau)d\tau \\ &= g(t) + \int_0^t H(t-\tau)u_k(\tau)d\tau, \end{aligned} \tag{9}$$

where

$$g(t) = Ce^{At}x^{o},$$
$$H(t-\tau) = Ce^{A(t-\tau)}B. \tag{10}$$

Definition 1 (for example, see [7])

A linear time-invariant system described by Eq.(6) is said to be strictly positive, if for any $T > 0$ and any input $u(t)$, $t \in [0, T]$, the following inequality is satisfied with some constant $\alpha > 0$:

$$\int_0^t \int_0^t u^T(t)H(t-\tau)u(\tau)d\tau dt \geqq \alpha \int_0^t u^T(t)u(t)dt. \tag{11}$$

In what follows, we denote the L^2-norm of vector-valued functions $u(t)$, $t \in [0, T]$, by

$$\|u\| = [\int_0^T u^T(t)u(t)dt]^{1/2}. \tag{12}$$

Now we are in a position to state:

Theorem 1 Suppose that the linear time-invariant system is strictly positive and the gain matrix of C^o-type betterment process is set in the following way:

$$\Phi = \gamma I \quad (I: m \times m \text{ identity matrix }) \tag{13}$$

with a sufficiently small $\gamma > 0$. Then, the C^o-type betterment process is convergent in the sense that there is a number $0 \leqq \rho < 1$ such that

$$\|e_{k+1}\| \leqq \rho \|e_k\| , \tag{14}$$

7

and hence

$$\|e_k\| \leqq \rho^k \|e_o\| \longrightarrow 0 \quad \text{as } k \to \infty. \tag{15}$$

Proof It follows from Eqs.(2), (7), and (13) that

$$y_{k+1}(t) = g(t) + \int_o^t H(t-\tau)\{u_k(\tau) + \gamma e_k(\tau)\}d\tau$$

$$= y_k(t) + \gamma \int_o^t H(t-\tau)e_k(\tau)d\tau. \tag{16}$$

Hence, from Eq.(3) it follows that

$$e_{k+1}(t) = e_k(t) - \gamma \int_o^t H(t-\tau)e_k(\tau)d\tau \tag{17}$$

which implies

$$\|e_{k+1}\|^2 = \|e_k\|^2 + \gamma^2\|w_k\|^2 - 2\gamma \int_o^T e_k^T(t)w_k(t)dt \tag{18}$$

where

$$w_k(t) = \int_o^t H(t-\tau)e_k(\tau)d\tau. \tag{19}$$

Here it is necessary to note that according to Parseval's equality in relation to the Fourier transform in function space $L^2[0, T]$, we have

$$\|w_k\|^2 = \lambda\{\int_o^T H^T(t)H(t)dt\}\|e_k\|^2 \leqq \lambda_o\|e_k\|^2 , \tag{20}$$

where $\lambda_o = \lambda\{X\}$ denotes the spectre radius of matrix $\{X\}$. Then, substituting Eq.(20) into Eq.(18) and noting the strict positivity of the system (see Eq.(11)), we get

$$\| e_{k+1} \|^2 \leqq (1 + \lambda_o \gamma^2 - 2\alpha\gamma) \| e_k \|^2. \tag{21}$$

Hence, choosing $\gamma > 0$ sufficiently small so that

$$\rho^2 = 1 + \lambda_o \gamma^2 - 2\alpha\gamma < 1, \tag{22}$$

we get

$$\| e_{k+1} \| \leqq \rho \, \| e_k \|, \qquad 0 \leqq \rho < 1. \tag{23}$$

This completes the proof.

IV. CONVERGENCE OF C^o-TYPE BETTERMENT PROCESS FOR LINEAR TIME-INVARIANT MECHANICAL SYSTEMS

In this section we will present a proof of convergence of a C^o-type betterment process for a class of linear time-invariant mechanical systems.

Suppose that the objective system is subject to a system of linear differential equations

$$R\ddot{x} + Q\dot{x} + Px = u, \tag{24}$$

where

$$u,\ x \in R^n, \qquad R,\ Q,\ P \in R^{n\times n}. \tag{25}$$

It is assumed that all coefficient matrices R, Q, and P are positive definite. In such a case, the system is often called a linear mechanical system. Suppose that the velocity vector x can be measured and set

$$y(t) = \dot{x}(t) \tag{26}$$

Given a desired output $y_d(t)$ over a finite time interval $t\in[0,T]$, a C^o-type betterment process yields

$$\begin{cases} R\ddot{x}_k + Q\dot{x}_k + Px_k = u_k, \\ y_k = \dot{x}_k, \\ u_{k+1} = u_k + \Phi(y_d - y_k). \end{cases} \tag{27}$$

We assume that at every operation trial initial values of x_k and $\dot{x}_k$ are set the same, namely,

$$x_k(0) = x^o, \qquad \dot{x}_k(0) = \dot{x}^o \tag{28}$$

In addition, we assume that each component of $u_o(t)$ is continuous and each component of $y_d(t)$ is continuously differentiable, namely,

$$u_o(t) \in C[0,T] \quad \text{and} \quad y_d(t) \in C^1[0,T]. \tag{29}$$

Then, it follows from Eq.(27) that

$$\begin{aligned} R(\ddot{x}_{k+1} - \ddot{x}_k) + Q(\dot{x}_{k+1} - \dot{x}_k) + P(x_{k+1} - x_k) \\ = u_{k+1} - u_k = \Phi e_k \end{aligned} \tag{30}$$

where we put

$$e_k = y_d - y_k. \tag{31}$$

Now, define

$$d_k = x_{k+1} - x_k \tag{32}$$

and note that Eq.(30) can be rewritten into

$$R\ddot{d}_k + Q\dot{d}_k + Pd_k = \Phi e_k \tag{33}$$

and the following relation holds:

$$\dot{d}_k = e_k - e_{k+1}. \tag{34}$$

From these equations it follows that

$$\begin{aligned} \int_0^t e_{k+1}^T(\tau)\Phi e_{k+1}(\tau)d\tau = \int_0^t e_k^T(\tau)\Phi e_k(\tau)d\tau + \int_0^t \dot{d}_k^T(\tau)\Phi\dot{d}_k(\tau)d\tau \\ -2\int_0^t e_k^T(\tau)\Phi\dot{d}_k(\tau)d\tau \end{aligned} \tag{35}$$

where Φ is assumed to be symmetric. In view of Eq.(33), $\dot{d}_k$ is regarded as the output when an input $u_k = \Phi e_k$ is given to the system defined by Eqs.(24) and (26). Hence, if the system were strictly positive, the C^o-type betterment process defined by Eq.(27) would be convergent. Unfortunately, the system defined by Eqs.(24) and (26) is not so in general. Nevertheless, it is possible to prove the following result:

<u>Theorem 2</u> Assume that $u_o(t)$ and $y_d(t)$ satisfy Eq.(29) together with

$$y_d(0) = \dot{x}_k(0) = \dot{x}^o, \qquad k=0,1,2, \ldots \tag{36}$$

and gain matrix Φ is symmetric positive definite and satisfies

$$Q > \Phi > 0 \tag{37}$$

($Q-\Phi$ is positive definite). Then the C^o-type betterment process is convergent in the sense that

$$e_k(t) \rightarrow 0 \quad \text{as} \quad k \rightarrow \infty \tag{38}$$

uniformly in $t \in [0,T]$.

The proof of this theorem is given in Appendix, since it is rather elaborative.

Finally we remark that Theorem 2 is valid even if P is non-negative definite but the positive definiteness of R and Q is cruciel. In practical situations, if the smallest eigenvalue of Q is not large enough then construction of a negative velocity feedback loop may be effective in accelerating the speed of convergency for the betterment process.

V. CONVERGENCY CONDITIONS ON C^1-TYPE and MIXED-TYPE BETTERMENT PROCESSES

When the C^1-type betterment process is used, the iterative learning process converges for a more general class of dynamical systems. At first, suppose that the objective system is governed by

$$\begin{cases} \dot{x} = f(t,x) + Bu , & (39a) \\ y = Cx , & (39b) \end{cases}$$

where

$$x,f \in R^n, \quad u,y \in R^m, \quad B \in R^{n\times m}, \quad C \in R^{m\times n}, \tag{40}$$

We assume that f is Lipschitz continuous, namely,

$$\|f(t,x_1)-f(t,x_2)\|_\infty \leqq \alpha(t)\|x_1-x_2\|_\infty \tag{41}$$

for any $t \in [0,T]$ and any pair (x_1,x_2) in a certain domain $\Omega\times\Omega \subseteq R^n\times R^n$, where $\alpha(t)$ is a piecewise continuous function and

$$\|x\|_\infty = \max_{i=1,\cdots,n} |x_i| \tag{42}$$

By symbol $\|A\|_\infty$ we denote the norm of matrix A induced by vector norm $\|x\|_\infty$. Also we define a function norm $\|e\|_\lambda$ for an n-vector-valued function defined over $t \in [0,T]$ as

$$\|e\|_\lambda = \sup_{0\leqq t\leqq T} \{e^{-\lambda t}\|e(t)\|_\infty\}, \tag{43}$$

where $\lambda>0$ is an appropriately chosen fixed number.

Now, assume that at each operation the system is set at the same initial condition, namely,

$$x_k(0) = x^o , \qquad k=0,1, \ldots \tag{44}$$

Then, the following theorem holds:

<u>Theorem 3</u> If the following three conditions are satisfied, then the C^1-type betterment process is convergent in a sense that $y_k(t) \to y_d(t)$ as $k\to\infty$ uniformly in $t \in [0,T]$:

1) $\|I - CB\Gamma\|_\infty < 1$, (45)

2) $u_o(t), y_d(t) \in C^1[0,T]$, (46)

3) $y_d(0) = Cx^o$. (47)

The details of proof is given in our previous paper [3]. Here, we repeat only key points. Firstly we note that

$$\dot{e}_{k+1}(t) = (I - CB\Gamma)\dot{e}_k(t) + C[f(t,x_k(t) - f(t,x_{k+1}(t))], \tag{48}$$

which yields

$$\|\dot{e}_{k+1}\|_\lambda \leqq \|I - CB\Gamma\|_\infty \|\dot{e}_k\|_\lambda + \alpha_o \|C\|_\infty \|x_k - x_{k+1}\|_\lambda , \tag{49}$$

where $\alpha_o = \max_{t\in[0,T]} \alpha(t)$.

It has been shown by applying Gronwall's lemma [8] that the second term of the right hand side is bounded from above by $\phi(\lambda)\|\dot{e}_k\|_\lambda$, where $\phi(\lambda)$ is a function of order λ^{-1}. Thus we have

$$\|\dot{e}_{k+1}\|_\lambda \leqq \{\|I-CB\Gamma\|_\infty + O(\lambda^{-1})\}\|\dot{e}_k\|_\lambda . \tag{50}$$

Finally, if we take λ large enough, then, according to condition 1) in Theorem 3, there is a constant ρ such that

$$\|\dot{e}_{k+1}(t)\|_\lambda \leqq \rho \|\dot{e}_k\|_\lambda, \quad 0 \leqq \rho < 1. \tag{51}$$

This implies

$$\dot{e}_k(t) \to 0 \quad \text{as} \quad k \to \infty \tag{52}$$

uniformly in $t \in [0,T]$ and, owing to $e_k(0)=0$ by condition 3, integration of Eq.(52) implies

$$e_k(t) \to 0 \quad \text{as } k \to \infty . \tag{53}$$

Now, consider the mixed-type betterment process defined by Eq.(5) under the same condition of Theorem 3. Then, similarly to Eq.(48), we obtain

$$\dot{e}_{k+1}(t) = (I-CB\Gamma)\dot{e}_k(t) - CB\Phi e_k(t) + C[f(t,x_k(t)-f(t,x_{k+1}(t))]. \tag{54}$$

Since it follows that

$$\begin{aligned}\|e_{k+1}\|_\lambda &= \sup_{t\varepsilon[0,T]} \int_o^t e^{-\lambda(t-\tau)}\|e^{-\lambda\tau}\dot{e}_k(\tau)\|_\infty d\tau \\ &= \sup_{t\varepsilon[0,T]} \{\int_o^t e^{-\lambda(t-\tau)}d\tau\}\cdot\|\dot{e}_k\|_\lambda \\ &= \frac{1-e^{-\lambda T}}{\lambda}\|\dot{e}_k\|_\lambda = O(\lambda^{-1})\cdot\|\dot{e}_k\|.\end{aligned} \tag{55}$$

In applying Gronwall's lemma for evaluation of the magnitude of the last term in Eq.(54), a term of $e_k(t)$ appears, too. Nevertheless, all these extra terms are bounded from above by $O(\lambda^{-1})\|\dot{e}_k\|_\lambda$. Hence, we eventually obtain the same expression as Eq.(50) and conclude:

Theorem 4 Under the same conditions as in Theorem 3, the mixed-type betterment process is convergent in a sense that $y_k(t) \to y_d(t)$ as $k\to\infty$ uniformly in $t\varepsilon[0,T]$.

The C^1-type betterment process betters the performance of next operation in the meaning of performance criterion, $\|\dot{e}_k\|_\lambda$. However, if the L^2-norm on interval $[0,T]$ is used alternatively, it is not clear whether the C^1-type (or mixed-type) betterment process improves the performance at every run or not.

As a matter of course, both C^1-type and mixed-type betterment processes are convergent if the objective system is governed by a linear time-invariant dynamical system described by Eq.(8) and the following condition is satisfied:

$$\| I - CB\Gamma \|_\infty < 1. \tag{56}$$

In addition to this, if the C^o-type betterment process is convergent for the same linear time-invariant system then another mixed-type betterment process defined by

$$u_{k+1} = u_k + (\alpha\Phi + (1-\alpha)\Gamma\frac{d}{dt})e_k \tag{57}$$

is convergent. That is, the following relation holds:

$$y_k^{m)} = \alpha y_k^{o)} + (1-\alpha)y_k^{1)} \tag{58}$$

where $y_k^{m)}$, $y_k^{o)}$, $y_k^{1)}$ denote the output trajectory when the mixed-type, C^o-type, or C^1-type betterment process is respectively used under the same conditions on $u_o(t)$ and $y_d(t)$. Eqs.(57) and (58) suggests us an idea that it may be possible to deduce a better iterative learning process by choosing the parameter α appropriately on the basis of operation data at a few early trials.

VI. DISCUSSIONS ON APPLICABILITY OF LEARNING CONTROL METHODS FOR ROBOT MANIPULATORS

There are two approaches for clarifying the potentialities of proposed learning control methods in application to robot manipulators. It is well known (for example, see [9], [10], [11]) that the dynamics of serial-link manipulators with n degrees of freedom is described by

$$(J_o + H(q))\ddot{q} + f(q,\dot{q}) + g(q) = Ku, \tag{59}$$

where $q \varepsilon R^n$ is the vector of joint coordinates, H is a positive definite inertia matrix, J_o is a positive diagonal matrix representing inertial terms of internal load distribution of actuators, $f(q,\dot{q})$ is a vector-valued function of centrifugal, coriolis, and viscous frictional forces, g(q) is a vector due to gravity force, u is a vector of input voltages given to actuator servo-motors, and K is a diagonal gain matrix. At first we treat directly this nonlinear system representation, which may be rewritten into the state space form

$$\begin{cases} \dot{q} = p, \\ \dot{p} = -(J_o + H(q))^{-1}[f(q,p)+g(q)] + (J_o + H(q))^{-1}Ku. \end{cases} \tag{60}$$

Here we assume that the velocity vector $p=\dot{q}$ can be measured and thereby regarded as the output. Then, if we set

$$x^T = [q^T, p^T] , \tag{61}$$

the robot system may be expressed as

$$\begin{cases} \dot{x} = f(x) + B(x)u, \\ y = [0,I]x = Cx. \end{cases} \qquad (62)$$

Comparing this with Eq.(39), we note that the coefficient matrix B(x) in Eq.(62) is not constant but dependent on state x. However, it has been shown by the authors [2], [12] that for a desired velocity profile $y_d=\dot{q}_d(t)$ given over $t\varepsilon[0,T]$ the C^1-type betterment process converges if an initial input $u_o(t)$ is adequately chosen so that the corresponding output $y_o(t)$ lies in a neighborhood of $y_d(t)$ and, in addition, the following condition is satisfied:

$$\| I - (J_o + H(q))^{-1} K\Gamma \|_\infty < 1 \qquad (63)$$

for all q. Fortunately, it is possible to expect reasonably, due to the stable and robust structure of feedback laws [11], that a control law defined by

$$u_o(t) = K^{-1}g(q) + \bar{A}(q_d - q) + \bar{B}(\dot{q}_d - \dot{q}) \qquad (64)$$

gives rise to an output $\dot{q}(t)$ that remains in a neighborhood of $\dot{q}_d(t)$. Moreover, since in ordinary serial-link robots each entry of H(q) is a combination of trigonometric functions of only rotational components of q, there are two positive definite diagonal matrices such that

$$\Lambda_o \leqq H(q) \leqq \Lambda_1. \qquad (65)$$

Therefore, the key condition described by Eq.(63) is satisfied with a sufficient margin by choosing Γ so that it approximates $K^{-1}(J_o + \Lambda_1)$.

In case that a desired path is given and described in terms of task coordinates $y=(y_1, \ldots, y_m)$, it is necessary to modify the proposed learning scheme as follows:

$$u_{k+1}(t) = u_k(t) + J^T(q(t))\frac{d}{dt} e_k(t) . \qquad (66)$$

The detailed proof for the convergency of this learning process will be given in our subsequent paper [12].

The use of C^1-type betterment process for the system of Eq.(62) means that the differentiation of the velocity vector must be carried out. In practice, the velocity signal measured through a tacho-generator is often contaminated by noise. Hence, if the numerical differentiation does not work well in such a case, the use of C^o-type betterment process can be recommended by the following argument. At first we note that given a desired trajectory $y_d=q_d$ on [0,T], Eq.(59) is rewritten into the following linear differential equation in terms of deviation vectors $z_k=y_k-y_d$ and $v_k=u_k-u_d$:

$$(J_o + H(t))\ddot{z}_k + B(t)\dot{z}_k + A(t)z_k = h(t) + Kv_k(t), \qquad (67)$$

where u_d means a desired input that realizes the desired output $y_d(t)$, $H(t)=H(q_d(t))$, and $B(t)$, $A(t)$, and $h(t)$ are appropriate matrix-valued and vector-valued functions, respectively. Obviously the inertia matrix $J_o+H(t)$ is positive definite by definition, but other coefficient matrices are not necessarily positive definite. However, if a linear position and velocity feedback loop of form

$$v_k = u_k - (\bar{A}z_k + \bar{B}\dot{z}_k) \qquad (68)$$

is constructed locally as implicitly adopted in servo loops of onventional industrial robots, Eq.(67) is reduced to

$$(J_o + H(t))\ddot{z}_k + (B(t)+B)\dot{z}_k + (A(t)+A)z_k = h(t) + Ku_k \ , \tag{69}$$

here $B=K\bar{B}$ and $A=K\bar{A}$. Then both coefficient matrices of the elocity term and position term become positive definite by choosing and $\bar{B}$ adequately. Differing from Eq.(24), Eq.(69) is time-arying. However, it can be shown (see our separate paper [13]) hat the C^o-type betterment process converges provided each diagonal lement of A and B are sufficiently large and $u_o(t)$ and $y_d(t)$ are ufficiently smooth.

Finally we mention that experimental results demonstrate the ffectiveness of both C^o-type and C^1-type betterment processes in ase of motion control of robot manipulators ([2],[4]) and further n case of force or hybrid (position and force) controls ([13],[14]).

CONCLUSIONS

Three types of iterative learning control scheme for linear and nonlinear dynamical systems have been proposed. It has been shown that given a desired output over a finite timc-duration the first learning control process is convergent if the objective system is linear and strictly positive, or is a linear mechanical system. It has also been shown that both the second and third learning processes become convergent for a class of linear and nonlinear dynamical systems. Applicabilities of these metnods for dynamic controls of robot manipulators are also discussed. In parallel to these, some of experimental results will be presented in our concurrent paper [13].

REFERENCES

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering Operation of robots by learning," Journal of Robotic Systems, vol.1, No.2, pp.123-140, 1984.

[2] S. Arimoto, S. Kawamura, and F. Miyazaki, "Can mechanical robots learn by themselves?" Proc. of 2nd International Symposium of Robotic Research, Kyoto, Japan, August 1984.

[3] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of dynamic systems by learning: A new control theory for servomechanism or mechatronics systems," Proc. 23rd IEEE CDC, Las Vegas, Nevada, Dec. 1984.

[4] S. Kawamura, F. Miyazaki, and S. Arimoto, "Iterative learning control for robotic systems," Proc. of IECON'84, Tokyo, Japan, Oct. 1984.

[5] F. Rosenblatt, Principles of Neurodynamics, Perceptrons and the Theory of Brain Mechanics. Washington: Spartan Books Co., 1961.

[6] Ya. Z. Tsypkin, Adaptation and Learning in Automatic Systems. New York: Academic Press, 1971.

[7] C. J. Harris and J. M. E. Valenca, The Stability of Input-Output Dynamical Systems, New York: Academic Press, 1983.

[8] J. K. Hale, Ordinary Differential Equations. New York: Interscience, 1969.

[9] R. P. Paul, Robot Manipulators. Cambridge: The MIT Press, 1981.

10] P. Vukobratovic, Scientific Fundamentals of Robotics 1 and 2. Berlin: Springer-Verlag, 1982.

[11] S. Arimoto and F. Miyazaki, "Asymptotic stability of feedback control laws for robot manipulators," submitted to the Symposium on Robot Control '85, Barcelona, Spain, Nov. 1985.

[12] S. Arimoto, S. Kawamura, and F. Miyazaki, "Mathematical theory of learning controls," to be presented to The 4th Yale Workshop on Applications of Adaptive Systems Theory, New Haven, CT, May 1985.

[13] S. Kawamura, F. Miyazaki and S. Arimoto, "Applications of learning methods for dynamic controls of robot manipulators," to be presented to the 24th IEEE CDC, Fort Lauderdale, Florida, Dec. 1985.

[14] S. Kawamura, F. Miyazaki, and S. Arimoto, "Hybrid position/force control of manipulators based on learning method," submitted to the '85 International Conference on Advanced Robotics, Tokyo, Japan, Sept. 1985.

APPENDIX

Proof of Theorem 2 It is evident from Eqs.(33) and (28) that

$$\int_0^t e_k^T(\tau)\Phi\dot{d}_k(\tau)d\tau = \int_0^t [R\ddot{d}_k(\tau) + Q\dot{d}_k(\tau) + Pd_k(\tau)]^T\dot{d}_k(\tau)d\tau$$

$$= \frac{1}{2}[\dot{d}_k^T(t)R\dot{d}_k(t) + d_k^T(t)Pd_k(t)] + \int_0^t \dot{d}_k^T(\tau)Q\dot{d}_k(\tau)d\tau. \quad \text{(A-1)}$$

Substitution of this into Eq.(35) yields

$$\int_0^t e_{k+1}^T(\tau)\Phi e_{k+1}(\tau)d\tau = \int_0^t e_k^T(\tau)\Phi e_k(\tau)d\tau - \int_0^t \dot{d}_k^T(\tau)(Q-\Phi)\dot{d}_k(\tau)d\tau$$

$$- \frac{1}{2}[\dot{d}_k^T(t)R\dot{d}_k(t) + d_k^T(t)Pd_k(t)]. \quad \text{(A-2)}$$

This holds for any t in [0,T]. It is now important to note that Eq.(29) implies $x_0(t) \in C^2[0,T]$, $e_0(t) \in C^1[0,T]$, and hence $d_0(t) \in C^3[0,T]$, for its second derivative is in $C^1[0,T]$ according to Eq.(33). Therefore we have

$$R\dddot{d}_0 + Q\ddot{d}_0 + P\dot{d}_0 = \Phi\dot{e}_0 \quad \text{(A-3)}$$

with $\ddot{d}_0(0)=0$, which is due to Eqs.(28), (38) and (33) for k=0. Further note that $x_1(t)=d_0(t)+x_0(t)$ and hence $x_1(t) \in C^2[0,T]$, $e_1(t) \in C^1[0,T]$, and $d_1(t) \in C^3[0,T]$. In such a manner, it is obvious from mathematical induction that $d_k(t) \in C^3[0,T]$ and

$$R\dddot{d}_k + Q\ddot{d}_k + P\dot{d}_k = \Phi\dot{e}_k \quad \text{(A-4)}$$

with $\ddot{d}_k(0)=0$. Then, the same reasoning as Eq.(A-2) is derived induces the following:

$$\int_0^t \dot{e}_{k+1}^T(\tau)\Phi\dot{e}_{k+1}(\tau)d\tau = \int_0^t \dot{e}_k^T(\tau)\Phi\dot{e}_k(\tau)d\tau - \int_0^t \ddot{d}^T(\tau)(Q-\Phi)\ddot{d}_k(\tau)d\tau$$

$$- \frac{1}{2}[\ddot{d}_k^T(t)R\ddot{d}_k(t) + \dot{d}_k^T(t)P\dot{d}_k(t)]. \quad \text{(A-5)}$$

Let

$$a_k(t) = \int_0^t e_k^T(\tau)\Phi e_k(\tau)d\tau \ , \quad \text{(A-6)}$$

$$b_k(t) = \int_0^t \dot{e}_k^T(\tau)\Phi \dot{e}_k(\tau)d\tau \ . \quad \text{(A-7)}$$

Since Φ, $Q-\Phi$, and R are all positive definite, both $a_k(t)$ and $b_k(t)$ are monotonically non-increasing with increasing k and bounded from below. Hence there exist two numbers $a_o \geqq 0$ and $b_o \geqq 0$ such that

$$a_k(t) \to a_o \quad \text{and} \quad b_k(t) \to b_o \quad \text{as} \quad k \to \infty \quad \text{(A-8)}$$

uniformly in $t \in [0,T]$. This implies, in view of Eqs.(A-2) and (A-5), that

$$\dot{d}_k(t) \to 0 \quad \text{and} \quad \ddot{d}_k(t) \to 0 \quad \text{as} \quad k \to \infty \quad \text{(A-9)}$$

uniformly in $t \in [0,T]$. In addition, since $d_k(0)=0$, Eq.(A-9) implies

$$d_k(t) \to 0 \quad \text{as} \quad k \to \infty \quad \text{(A-10)}$$

uniformly in $t \in [0,T]$. In view of Eq.(33), Eqs.(A-9) and (A-10) give rise to the conclusion that

$$e_k(t) \to 0 \quad \text{as} \quad k \to \infty \quad \text{(A-11)}$$

uniformly in $t \in [0,T]$.

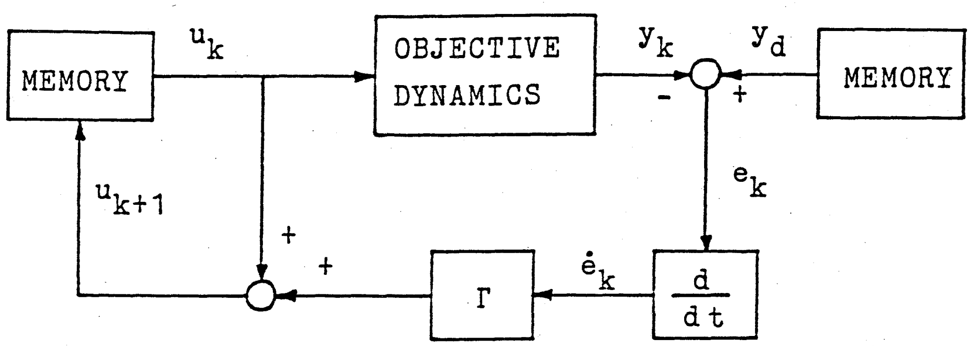

Fig.1 C^1-type betterment process

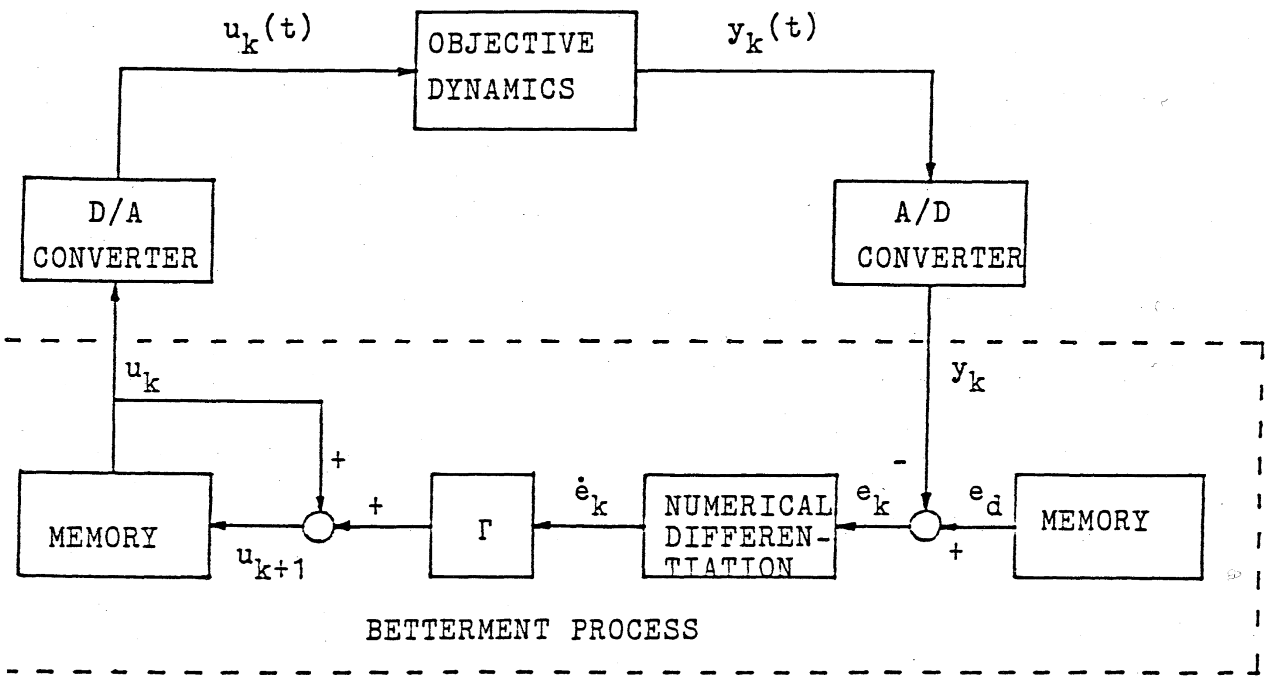

Fig.2 Implementation of betterment process

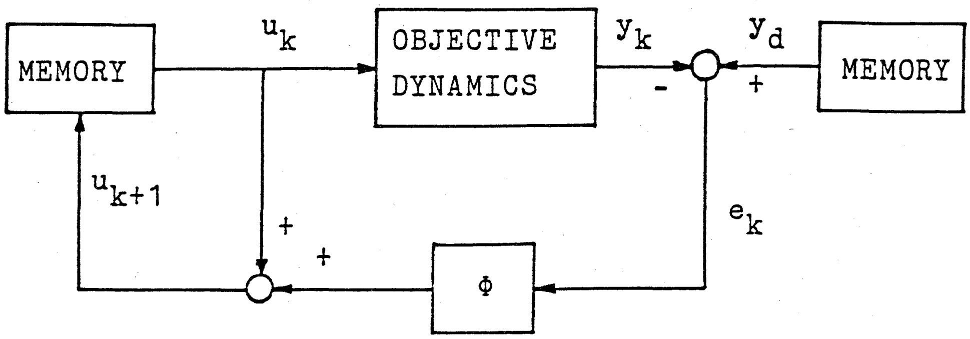

Fig.3 C^o-type betterment process

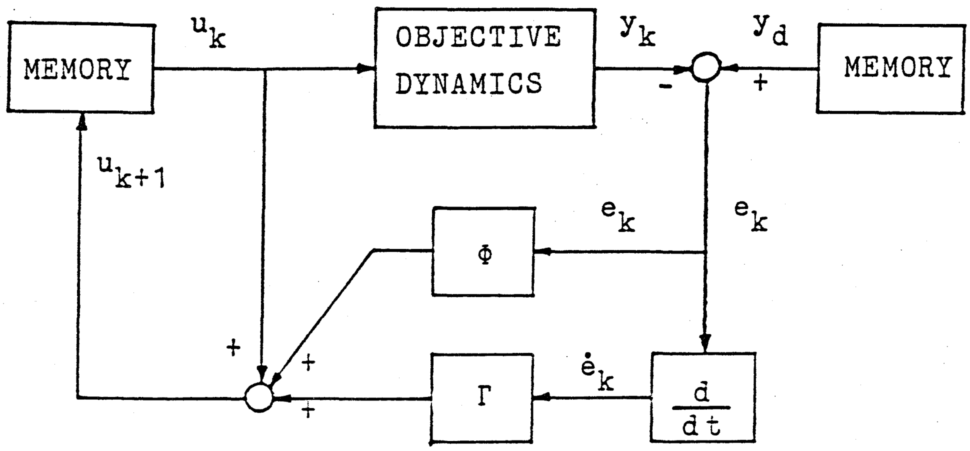

Fig.4 Mixed-type betterment process